

# Inflationary Attractor in Braneworld Scenario

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We demonstrate the attractor behavior of inflation driven by a scalar field or a tachyon field in the context of recently proposed four-dimensional effective gravity induced on the world-volume of a three-brane in five-dimensional Einstein gravity, and we obtain a set of exact inflationary solutions. Phase portraits indicate that an initial kinetic term decays rapidly and it does not prevent the onset of inflation. The trajectories more rapidly reach the slow rolling curve in braneworld scenario than in the standard cosmology.

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## I. INTRODUCTION

Recent developments in string theory and its extension M-theory have suggested that the standard model particles are confined on a hypersurface (called brane) embedded in a higher dimensional space (called bulk). Only gravity and other exotic matter such as the dilaton can propagate in the bulk [1]. Our universe may be such a brane-like object. In the braneworld scenario, constraints on the size of extra dimensions become weaker because the standard model particles propagate only in three spatial dimensions. Arkani-Hamed, Dimopoulos and Dvali considered a flat bulk geometry in (4+d)-dimensions, in which  $d$  dimensions are compact with radius  $R$  (toroidal topology) [2]. Considerable progress was made by Randall and Sundrum, who considered non-flat, i.e. warped bulk geometries [3]. In their first paper, they proposed a mechanism to solve the hierarchy problem by a small extra dimension. In their second paper, the braneworld with a positive tension was investigated. The effective Einstein equations and the generalized Friedmann equation on the brane were derived [4, 5]. Since the energy density of the brane appears quadratically in the new Friedmann equation in contrast with the linear behaviour of the usual equation, the scenario has interesting cosmological implications, in particular, the prospects of inflation are enhanced on the brane [6]. Modifications to the Friedmann constraint equation lead to a faster Hubble expansion at high energies and a more strongly damped evolution of the scalar field. This assists slow rolling inflation, enhances the amount of inflation obtained in any given model, and drives the perturbations towards an exactly scale-invariant Harrison-Zel'dovich spectrum [6]. Recently, several authors have investigated the cosmological implications of a tachyon rolling down its potential, which have not been derived from string theory, to its

ground state [7, 8]. Sufficient inflation driven by the rolling tachyon can be achieved in the context of the braneworld scenario [9].

To date a question which has not yet been addressed in the literature on inflation in the braneworld scenario is the issue of constraints on the phase space of initial conditions for inflation. If inflation is to be truly predictive, the evolution when the scalar field is at some given point on the potential has to be independent of the initial conditions. Otherwise, any result, such as the amplitude of density perturbations, would depend on the unknowable initial conditions. However, the scalar wave equation is a second-order equation, implying that  $\phi$ , in principle, can take on any value anywhere on the potential, and so, there certainly is not a unique solution at each point on the potential. Therefore inflation can be predictive only if the solutions exhibit an attractor behavior, where the differences between solutions of different initial conditions rapidly vanish. In the standard cosmology the inflationary equations do indeed possess this vital property [10]. In the braneworld scenario, it is interesting and necessary to investigate the initial conditions for inflation. In this paper, using the Hamilton-Jacobi formalism, we show that inflation driven by a scalar field or a tachyon field on the brane possesses the attractor behavior and obtain a set of exact inflationary solutions. As examples we plot the trajectories in the phase space numerically. Phase portraits indicate that an initial kinetic term decays rapidly and it does not prevent the onset of inflation. Moreover, we find that the trajectories more rapidly reach the slow rolling curve in the braneworld scenario than in the standard cosmology.

## II. MODIFIED FRIEDMANN EQUATION IN THE BRANEWORLD SCENARIO

In the five-dimensional braneworld scenario [3], the four-dimensional Einstein equations induced on the brane

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can be written as [4]

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \kappa_4^2 T_{\mu\nu} + \kappa_5^4 \pi_{\mu\nu} - E_{\mu\nu}, \quad (1)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor of matter on the brane,  $\pi_{\mu\nu}$  is a tensor quadratic in  $T_{\mu\nu}$ ,

$$\pi_{\mu\nu} = \frac{1}{12} TT_{\mu\nu} - \frac{1}{4} T_{\mu\eta} T_\nu^\eta + \frac{1}{8} g_{\mu\nu} T_{\eta\lambda} T^{\eta\lambda} - \frac{1}{24} T^2 g_{\mu\nu}, \quad (2)$$

and  $E_{\mu\nu}$  is a projection of the five-dimensional Weyl tensor, describing the effect of bulk graviton degrees of freedom on brane dynamics. The effective cosmological constant  $\Lambda_4$  on the brane is determined by the five-dimensional bulk cosmological constant  $\Lambda_5$  and the brane tension  $\sigma$  as

$$\Lambda_4 = \frac{\kappa_5^2}{2} \left( \Lambda_5 + \frac{\kappa_5^2}{6} \sigma^2 \right), \quad (3)$$

and the four-dimensional Planck scale is given by

$$\kappa_4^2 = \frac{\kappa_5^2}{6} \sigma. \quad (4)$$

In a spatially flat Friedmann-Roberson-Walker (FRW) model the Friedmann equation on the brane becomes [5]:

$$H^2 = \frac{\Lambda_4}{3} + \frac{\kappa_4^2}{3} \rho \left( 1 + \frac{\rho}{2\sigma} \right) + \frac{\mu}{a^4}. \quad (5)$$

where  $\mu$  is an integration constant arising from  $E_{\mu\nu}$ , and thus transmitting bulk graviton influence onto the brane. This term is called the dark radiation term, which can be obtained from a full analysis of the bulk equations. In the following we will assume that  $\mu = 0$  and  $\Lambda_4 = 0$ , which indicates that these is a fine-tuning  $\Lambda_5 = -\frac{\kappa_5^2}{6} \sigma^2$ . If the brane tension  $\sigma$  is much below the energy scale on the brane, then the new term in  $\rho^2$  is dominant in the modified Friedmann equation. Hence, the term in  $\rho$  is negligible during inflation in early universe.

### III. ATTRACTOR BEHAVIOR FOR STANDARD INFLATON

We consider a scalar field  $\phi$  confined on the brane with self-interaction potential  $V(\phi)$ . The evolution equation of the field is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (6)$$

subject to the modified Friedmann constraint

$$H^2 = \frac{\kappa_4^2}{6\sigma} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right)^2. \quad (7)$$

One can derive a very useful alternative form of these equations by using the scalar field as a time variable [11]. This requires that  $\phi$  does not change sign during inflation. Without loss of generality, we can choose  $\dot{\phi} > 0$

throughout. If this is not satisfied, it can be brought about by redefining  $\phi \rightarrow -\phi$ . Differentiating Eq.(7) with respect to  $t$  and using Eq.(6) gives

$$\dot{H} = -\frac{3\kappa_4}{\sqrt{6\sigma}} H \dot{\phi}^2. \quad (8)$$

We may divide each side by  $\dot{\phi}$  to eliminate the time dependence in the Friedmann equation, obtaining

$$\dot{\phi} = -\frac{\sqrt{6\sigma}}{3\kappa_4} \frac{H'(\phi)}{H(\phi)}, \quad (9)$$

which gives the relation between  $\phi$  and  $t$ . This allows us to write the Friedmann equation in the first-order form

$$\frac{\sigma}{3\kappa_4^2} \left[ \frac{H'(\phi)}{H(\phi)} \right]^2 - \frac{\sqrt{6\sigma}}{\kappa_4} H(\phi) = -V(\phi). \quad (10)$$

This new set of equations (9) and (10) is the Hamilton-Jacobi equations, which is normally more convenient than the Eqs.(6) and (7). It allows us to consider  $H(\phi)$ , rather than  $V(\phi)$ , as the fundamental quantity to be specified. Once  $H(\phi)$  has been specified, we immediately can obtain the corresponding potential from Eq.(10). Also, Eq.(9) gives the relation between  $\phi$  and  $t$ , which enables us to obtain  $H(t)$ , which, if it is desired, can be integrated to  $a(t)$ . Therefore, this is the most direct route to obtaining a large set of exact inflationary solutions.

As an example of using (9) and (10), let us consider a universe in the braneworld scenario with  $H(\phi) = \phi^{-\alpha}$ . Elementary algebra now gives the potential to be of the form

$$V(\phi) = \frac{\sqrt{6\sigma}}{\kappa_4} \phi^{-\alpha} - \frac{\sigma\alpha^2}{3\kappa_4^2} \phi^{-2}. \quad (11)$$

The corresponding evolution of  $\phi(t)$  is

$$\phi(t) = \left( \phi_0^2 + \frac{2\sqrt{6\sigma}\alpha}{3\kappa_4} t \right)^{\frac{1}{2}}. \quad (12)$$

Then we obtain

$$a(t) \propto \begin{cases} \left( 1 + \frac{4\sqrt{6\sigma}}{3\phi_0^2\kappa_4} t \right)^{\frac{3\kappa_4}{4\sqrt{6\sigma}}} & \alpha = 2 \\ \exp \left[ \frac{3\kappa_4}{\alpha(2-\alpha)\sqrt{6\sigma}} \left( \phi_0^2 + \frac{2\sqrt{6\sigma}\alpha}{3\kappa_4} t \right)^{1-\frac{\alpha}{2}} \right] & \alpha \neq 2. \end{cases} \quad (13)$$

Using the Hamilton-Jacobi equations (9) and (10), we demonstrate the attractor behavior in the braneworld scenario. Suppose  $H_0(\phi)$  is any solution to Eq.(10), which can be either inflationary or non-inflationary. Add to this a linear homogeneous perturbation  $\delta H(\phi)$ ; the attractor condition will be satisfied if it becomes small as  $\phi$  increases. For simplicity, we shall assume, discuss further below, that the perturbations do not reverse the sign of  $\dot{\phi}$ , though the result holds under more general circumstances. Substituting  $H(\phi) = H_0(\phi) + \delta H(\phi)$  into

Eq.(10) and linearizing it, we find that the perturbation obeys

$$H'_0 \delta H' \simeq \left( \frac{9\sqrt{6}\kappa_4}{2\sqrt{\sigma}} H_0 - \frac{3\kappa_4^2}{\sigma} V \right) H_0 \delta H, \quad (14)$$

which has the general solution

$$\delta H(\phi) = \delta H(\phi_i) \exp \left[ \int_{\phi_i}^{\phi} \left( \frac{3\sqrt{6}\kappa_4}{2\sqrt{\sigma}} \frac{H_0^2}{H'_0} + \frac{H'_0}{H_0} \right) d\phi \right], \quad (15)$$

where  $\delta H(\phi_i)$  is the value at some initial point  $\phi_i$ . We have used the fact that  $H_0$  is any solution to Eq.(10). Because  $H'_0$  and  $d\phi$  have opposing signs, the integrand within the exponential term is negative definite, and hence all linear perturbations do indeed die away. If there is an inflationary solution, all linear perturbations approach it at least exponentially fast as the scalar field rolls.

To study an explicit numerical computation of the phase space trajectories in the braneworld scenario, it is most convenient to rewrite the evolution equations (7) and (6) for  $H$  and  $\phi$  as a set of two first-order differential equations with two independent variables  $\phi$  and  $\dot{\phi}$

$$\frac{d\phi}{dt} = \dot{\phi}, \quad (16)$$

$$\frac{d\dot{\phi}}{dt} = -\frac{3\kappa_4}{\sqrt{6}\sigma} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] \dot{\phi} - V'(\phi). \quad (17)$$

Let us consider two different examples. In the first example, we consider a new inflation model in the braneworld scenario, with a potential of Coleman-Weinberg type

$$V(\phi) = \lambda \phi^4 \left( \ln \frac{\phi^2}{\varphi^2} - \frac{1}{2} \right) + \frac{1}{2} \lambda \varphi^4. \quad (18)$$

We choose different initial conditions in the range  $|\phi_0| < \varphi$  and  $|\dot{\phi}_0| < \sqrt{\lambda} \varphi^2$ , and we follow the evolution until the field begins to oscillate around the true vacuum at  $\pm\varphi$ . Figure 1 displays the trajectories in the  $(x, y)$  plane for this model in the braneworld scenario, where  $x$  and  $y$  are dimensionless coordinates

$$\phi = \varphi x, \quad \dot{\phi} = \sqrt{\lambda} \varphi^2 y, \quad t = \frac{1}{\sqrt{\lambda} \varphi} \eta, \quad y = \frac{dx}{d\eta}.$$

We find that an initial kinetic term decays rapidly and it does not prevent the onset of inflation. There is a curve that attracts most of the trajectories. This curve corresponds to the slow rolling solution. Moreover, comparing the phase portrait Figure 1 to Figure 2 reveals that the trajectories more rapidly reach the slow rolling curve in the braneworld scenario than in the standard cosmology. Assuming  $\lambda \sim 1$ ,  $\varphi \sim M_p$  and  $\varphi^4/\sigma \sim 10$ , the trajectory with  $\dot{\phi}_0 \sim 0.3 M_p^2$  reaches the slow rolling curve at  $\phi \sim 0.3 M_p$  in Figure 1 while at  $\phi \sim 0.5 M_p$  in Figure 2.

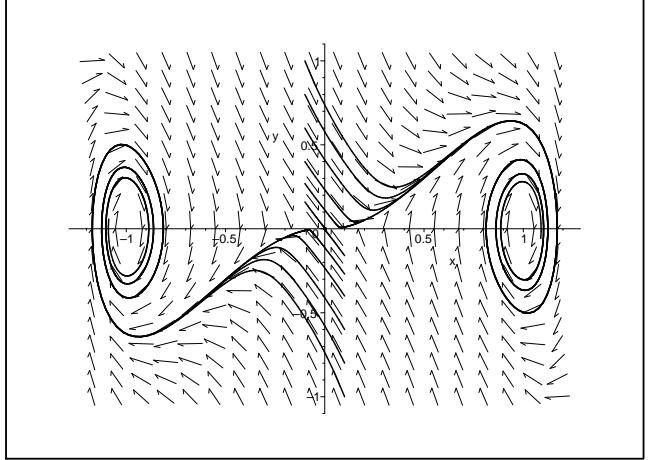


FIG. 1: Phase portrait for the scalar field with  $V(\phi) = \lambda \phi^4 (\ln \frac{\phi^2}{\varphi^2} - \frac{1}{2}) + \frac{1}{2} \lambda \varphi^4$  in rescaled coordinates  $(x, y)$  in the braneworld scenario.

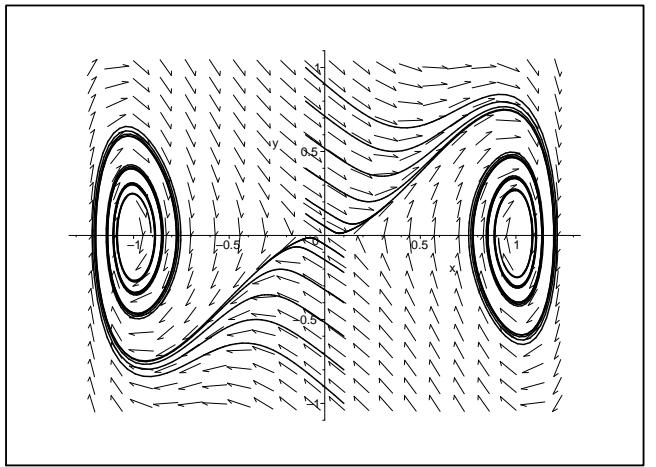


FIG. 2: Phase portrait for the scalar field with  $V(\phi) = \lambda \phi^4 (\ln \frac{\phi^2}{\varphi^2} - \frac{1}{2}) + \frac{1}{2} \lambda \varphi^4$  in rescaled coordinates  $(x, y)$  in the standard FRW scenario.

In the second example, we investigate the simplest chaotic inflation model on the brane driven by a scalar field with potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2. \quad (19)$$

All solutions to these two equations can be represented as trajectories in the two-dimensional phase space of  $\phi$  and  $\dot{\phi}$ . Figure 3 displays the trajectories in the  $(x, y)$  plane for this model, where  $x$  and  $y$  are dimensionless coordinates

$$\phi = mx, \quad \dot{\phi} = m^2 y, \quad t = \frac{1}{m} \eta, \quad y = \frac{dx}{d\eta}.$$

We see that there is a curve that attracts most of the trajectories. However, unlike the new inflation case where

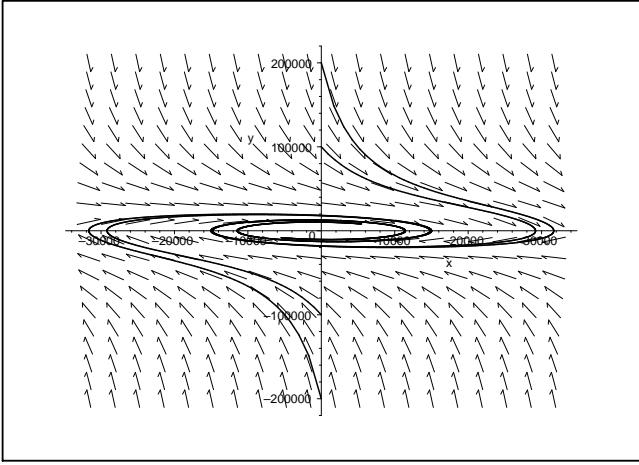


FIG. 3: Phase portrait for the scalar field with  $V(\phi) = \frac{1}{2}m^2\phi^2$  in rescaled coordinates  $(x, y)$  in the braneworld scenario.

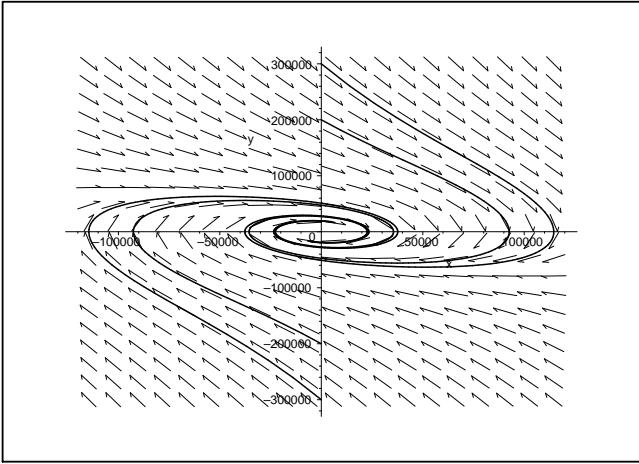


FIG. 4: Phase portrait for the scalar field with  $V(\phi) = \frac{1}{2}m^2\phi^2$  in rescaled coordinates  $(x, y)$  in the standard FRW scenario.

only a small part of it corresponds to inflation, in the chaotic case the attractor is the inflationary solution. Comparing Figure 3 to Figure 4, an initial kinetic term decays more rapidly in the braneworld scenario than that in standard cosmology. The COBE constraints require  $m \sim 10^{13}$  GeV [6]. Assuming  $V(\phi)/\sigma \sim 10$ , the initial kinetic term  $\dot{\phi}_0 \sim 2 \times 10^{31}$  GeV decays to zero at  $\phi \sim 3 \times 10^{17}$  GeV in Figure 3 while at  $\phi \sim 9 \times 10^{17}$  GeV in Figure 4.

#### IV. ATTRACTOR BEHAVIOR FOR TACHYONIC MATTER

According to Sen [7], the effective action of the tachyon field in the Born-Infeld form can be written as

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa_4^2} R + V(\phi) \sqrt{1 + \partial_\mu \phi \partial^\mu \phi} \right), \quad (20)$$

The rolling tachyon in a spatially flat FRW cosmological model can be described by a fluid with a positive energy density  $\rho$  and a negative pressure  $P$  given by

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad (21)$$

$$P = -V(\phi) \sqrt{1 - \dot{\phi}^2}, \quad (22)$$

The evolution equation of the tachyon field minimally coupled to gravity and the modified Friedmann constraint are

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'(\phi)}{V(\phi)} = 0, \quad (23)$$

$$H^2 = \frac{\kappa_4^2}{6\sigma} \frac{V^2(\phi)}{1 - \dot{\phi}^2}. \quad (24)$$

Differentiating Eq.(24) with respect to  $t$  and substituting it in Eq.(23) gives

$$\dot{H} = -3H^2\dot{\phi}^2. \quad (25)$$

We may divide each side by  $\dot{\phi}$  to eliminate the time dependence in the Friedmann equation, obtaining the Hamilton-Jacobi equations

$$\dot{\phi} = -\frac{H'(\phi)}{3H^2(\phi)}, \quad (26)$$

$$\frac{2\sigma}{3\kappa_4^2} \left[ \frac{H'(\phi)}{H(\phi)} \right]^2 - \frac{6\sigma}{\kappa_4^2} H^2(\phi) = -V^2(\phi). \quad (27)$$

Once  $H(\phi)$  has been specified, we can obtain the corresponding potential and exact inflationary solution using (26) and (27). For example,  $H(\phi) = \phi^{-\alpha}$  gives the potential

$$V(\phi) = \frac{\sqrt{\sigma}}{\kappa_4} \left( 6\phi^{-2\alpha} - \frac{2\alpha^2}{3}\phi^{-2} \right)^{\frac{1}{2}}. \quad (28)$$

The corresponding evolution of  $\phi(t)$  is

$$\phi(t) = \begin{cases} \phi_0 e^{\frac{2}{3}t} & \alpha = 2 \\ \left( \phi_0^{2-\alpha} + \frac{(2-\alpha)\alpha}{3}t \right)^{\frac{1}{2-\alpha}} & \alpha \neq 2. \end{cases} \quad (29)$$

Then we obtain the corresponding evolution of  $a(t)$

$$a(t) \propto \begin{cases} \left( 1 + \frac{1}{3}\phi_0^{-1}t \right)^3 & \alpha = 1 \\ \exp \left( -\frac{3}{4}\phi_0^{-2}e^{-\frac{4}{3}t} \right) & \alpha = 2 \\ \exp \left[ \frac{3}{2\alpha(1-\alpha)} \left( \phi_0^{2-\alpha} + \frac{2\alpha-\alpha^2}{3}t \right)^{\frac{2-2\alpha}{2-\alpha}} \right] & \text{others.} \end{cases} \quad (30)$$

Analogous to the preceding section we use the Hamilton-Jacobi formalism to analysis the attractor behavior of the tachyonic inflation in the braneworld scenario. Suppose  $H_0(\phi)$  is any solution to the full equation

of motion, Eq.(27), which can be either inflationary or non-inflationary. Consider a linear homogeneous perturbation  $\delta H(\phi)$ . It therefore obeys the linearized equation

$$H'_0 \delta H' \simeq \left( 18H_0^2 - \frac{3\kappa_4^2}{2\sigma} V^2 \right) H_0 \delta H, \quad (31)$$

which has the general solution

$$\delta H(\phi) = \delta H(\phi_i) \exp \left[ \int_{\phi_i}^{\phi} \left( \frac{9H_0^3}{H'_0} + \frac{H'_0}{H_0} \right) d\phi \right], \quad (32)$$

We have used the fact that  $H_0$  is any solution to Eq.(27). Since  $H'_0$  and  $d\phi$  have opposing signs, the integrand within the exponential term is negative definite, and hence all linear perturbations do indeed die away. That is, provided the potential is able to support inflation, the inflationary solutions all rapidly approach one another, with exponential rapidity once in the linear regime.

The tachyon potential  $V(\phi) \rightarrow 0$  as  $\phi \rightarrow \infty$ , but its exact form is not known at present. Sen has argued that the qualitative dynamics of string tachyons can be described by (20) with exponential potential [12], which is further investigated in support of this potential for the tachyon system [13]. The field equations (23) and (24) for tachyonic matter on the brane with the exponential potential

$$V(\phi) = V_0 e^{-\alpha\phi} \quad (33)$$

can be cast in the form

$$\frac{d\phi}{dt} = \dot{\phi}, \quad (34)$$

$$\frac{d\dot{\phi}}{dt} = (1 - \dot{\phi}^2) \left[ \alpha - \frac{3\kappa_4 V(\phi)}{\sqrt{6}\sigma} (1 - \dot{\phi}^2)^{-\frac{1}{2}} \phi \right]. \quad (35)$$

We choose different initial conditions  $\phi$  ( $0 \leq \phi$ ) and  $\dot{\phi}$  ( $0 \leq \dot{\phi} \leq 1$ ), and we follow the evolution with the exponential potential numerically. Figure 5 displays the trajectories in the  $(\phi, \dot{\phi})$  plane for this model. We see that there is a curve that attracts most of the trajectories. This curve corresponds to the slow rolling solution. A sufficient tachyon inflation consistent with the observational constraints in the braneworld scenario prefers the string coupling  $g_s \sim 10^{-16}$  and the string mass scale  $M_s \sim 10^{-7} M_p$  [9]. Thus the string energy density  $V_0 \sim 10^{-15} M_p^4$ . We find that the trajectory with  $\dot{\phi}_0 \sim 0.8 \times 10^{-7} M_p$  reaches the slow rolling solution at  $\phi \sim 0.7$  in Figure 5 in the braneworld scenario while at  $\phi \sim 1.2$  in Figure 6 in the standard cosmology, where we assume  $\alpha \sim 1/2$  and  $V_0/\sigma \sim 10$ . The numerical computation indicates that an initial kinetic term of tachyon field decays more rapidly in the braneworld scenario than in the standard cosmology if the brane tension remains sufficiently below the string energy density.

The effective tachyon field action given by Eq.(20) is a closed form expression incorporating all the higher power

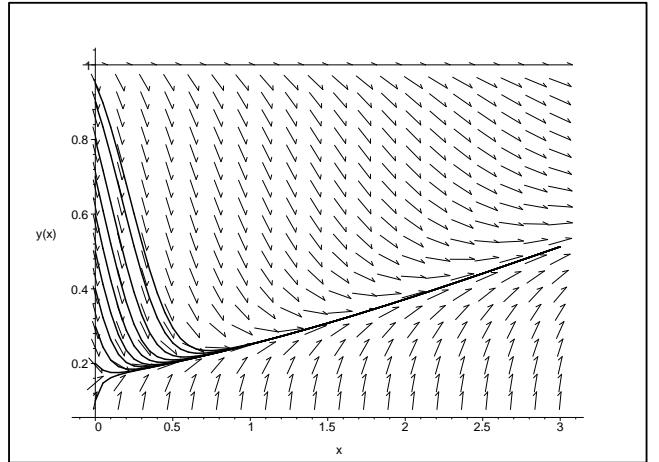


FIG. 5: Phase portrait for the tachyon field with  $V(\phi) = V_0 e^{-\alpha\phi}$  in rescaled coordinates  $(\phi, \dot{\phi})$  in the braneworld scenario.

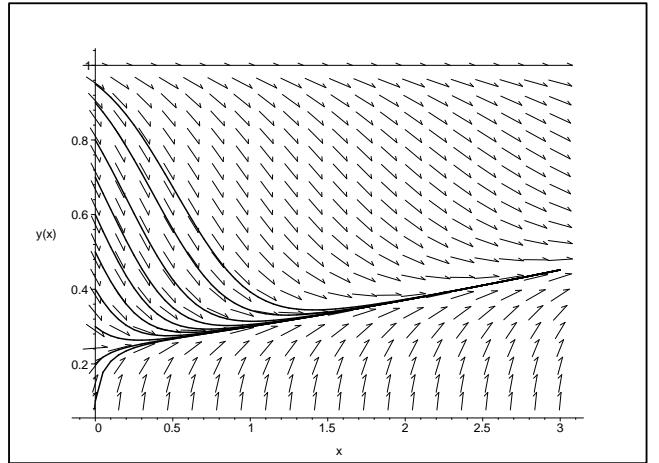


FIG. 6: Phase portrait for the tachyon field with  $V(\phi) = V_0 e^{-\alpha\phi}$  in rescaled coordinates  $(\phi, \dot{\phi})$  in the standard FRW scenario.

of  $\partial_\mu \phi$ . It is impossible to obtain a canonical kinetic term via a field redefinition. The momentum conjugate to  $\phi$  is given by

$$\Pi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \dot{\phi}. \quad (36)$$

Figure 7 and Figure 8 display the trajectories in the  $(\phi, \Pi)$  plane for this model in the braneworld scenario and in the standard FRW scenario respectively, which are consistent with Figure 5 and Figure 6. Comparing Figure 5 to Figure 7, when  $\phi \rightarrow \infty$ , the kinetic term  $\phi \rightarrow 1$  and the momentum density  $\Pi \rightarrow 0$  since  $V(\phi) \rightarrow 0$ .

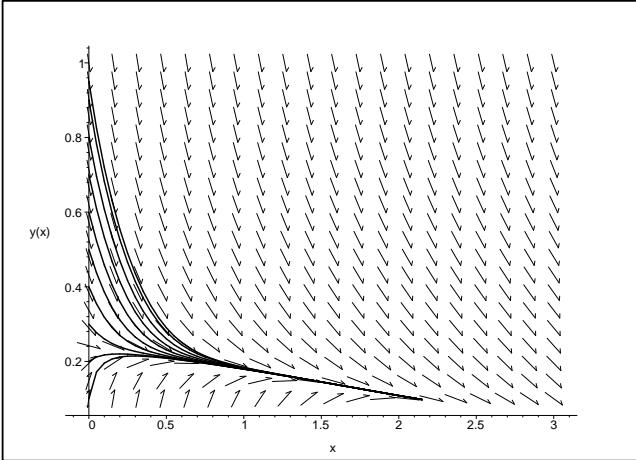


FIG. 7: Phase portrait for the tachyon field with  $V(\phi) = V_0 e^{-\alpha\phi}$  in rescaled coordinates  $(\phi, \Pi)$  in the braneworld scenario.

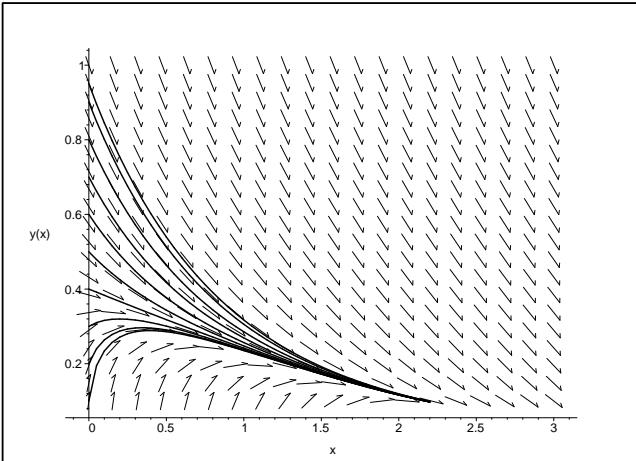


FIG. 8: Phase portrait for the tachyon field with  $V(\phi) = V_0 e^{-\alpha\phi}$  in rescaled coordinates  $(\phi, \Pi)$  in the standard FRW scenario.

## V. CONCLUSIONS AND DISCUSSIONS

We have derived the Hamilton-Jacobi equations of the inflation driven by a scalar field or a tachyon field, which is confined on a three-dimensional brane in five-dimensional Einstein gravity, and have obtained a set of exact inflationary solutions. To demonstrate the attractor behavior, we use the Hamilton-Jacobi formalism, which greatly simplifies the analysis. We restrict ourselves to linear homogeneous perturbations, which is all that is needed because, classically at least, inflation does indeed generate large smooth patches. If the perturbation is nonlinear, then the solution is made more complicated but, because the full equation is only first order, it is easy to see that solutions are compelled to approach one another regardless of whether the perturbation is lin-

ear or not. For simplicity, we also assume that the perturbations do not reverse the sign of  $\dot{\phi}$ , which can matter only if the perturbation takes the field over the top of a maximum in the potential because otherwise it will simply roll up, reverse its direction, and pass back down through the same point, where it can be regarded as a perturbation on the original solution with the same sign of  $\dot{\phi}$ .

Moreover, we have investigated two typical models for the scalar field: the new inflation with the Coleman-Weinberg potential and the chaotic inflation with the simplest potential, and have investigated the tachyon inflation on the brane with the exponential potential. The phase space trajectories indicate that an initial kinetic term decays rapidly and it does not prevent the onset of inflation, that is, inflation on the brane possesses the attractor behaviour. In phase space there is a curve that attracts most of the trajectories. This curve corresponds to the slow rolling solution. Our numerical results show that the initial kinetic terms more rapidly decay and so the trajectories more rapidly reach the inflationary solution in the braneworld scenario than in the standard cosmology. At low energies,  $\rho \ll \sigma$ , the modified Friedmann equation (5) reduces to the standard form since the term quadratic in the energy density is negligible. However at high energies,  $\rho \gg \sigma$ , the term quadratic in the energy density dominates the modified Friedmann equation. The effect of the modified Friedmann equations (7) and (24) at high energies is to increase the Hubble parameter by the new factor  $\rho/2\sigma$ . Thus the friction term  $3H\dot{\phi}$  in the evolution equation of the field (6) or (23) becomes larger, which leads to more rapid decay of the initial kinetic term. Thus brane effects widen the range of the initial conditions for successful inflation.

It is interesting to investigate cosmological evolutions in models where the effective potential  $V(\phi) = V_0 + \frac{1}{2}m^2\phi^2$  may become negative for some values of the field  $\phi$  in the braneworld scenario. Phase portraits of such theories in space of variables  $(\phi, \dot{\phi}, H)$  are different from phase portraits in the standard usual cosmological scenario [14]. For a flat universe  $k = 0$  with a negative potential  $V_0 < 0$ , all trajectories in the three-dimensional phase space  $(\phi, \dot{\phi}, H)$  are located at a paraboloid in contrast with a hyperboloid of one sheet in the standard cosmological scenario. Trajectories both in the expanding universe region ( $H > 0$ ) and in the contracting universe region ( $H < 0$ ) spiral in towards the center in two-dimensional projection  $(\phi, \dot{\phi})$  of the flat universe hypersurface for this model. However, there appears a wormhole connecting the expanding and contracting branches in the standard cosmological scenario [14]. In the braneworld scenario, cosmological evolution with negative potential may have more fruitful phenomena, which is worth studying further.

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- [1] P.Horava and E.Witten, Nucl.Phys. **B460** (1996) 506, hep-th/9510209.
- [2] N.Arkani-Hamed, S.Dimopoulos and G.Dvali, Phys.Lett. **B429** (1998) 263;  
I.Antoniadis, N.Arkani-Hamed, S.Dimopoulos and G.Dvali, Phys.Lett. **B436** (1998) 257.
- [3] L.Randall and R.Sundrum, Phys.Rev.Lett. **83** (1999) 3370, hep-ph/9905221;  
L.Randall and R.Sundrum, Phys.Rev.Lett. **83** (1999) 4690, hep-ph/9906064.
- [4] T.Shiromizu, K.Maeda and M.Sasaki, Phys.Rev. **D62** (2000) 024012, gr-qc/9910076.
- [5] P.Binetruy, C.Deffayet, U.Ellwanger and D.Langlois, Phys.Lett. **B477** (2000) 285, hep-ph/9910219.
- [6] R.Maartens, D.Wands, B.A.Bassett and I.P.C.Heard, Phys.Rev. **D62** (2000) 041301, hep-ph/9912464.
- [7] A.Sen, JHEP **0204** (2002) 048, hep-th/0203211;  
A.Sen, JHEP **0207** (2002) 065, hep-th/0203265.
- [8] G.W.Gibbons, Phys.Lett. **B537** (2002) 1, hep-th/0204008;  
M.Fairbairn and M.H.Tytgat,Phys.Lett. **B546** (2002) 1, hep-th/0204070;  
G.Shiu and I.Wasserman, Phys.Lett. **B541** (2002) 6, hep-th/0205003;
- [9] Y.S.Piao, R.G.Cai, X.M.Zhang and Y.Z.Zhang, Phys.Rev. **D66** (2002) 121301;  
Y.S.Piao, Q.G.Huang, X.M.Zhang and Y.Z.Zhang, Phys.Lett. **B570** (2003) 1;  
Y.S.Piao and Y.Z.Zhang, hep-th/0307074.
- [10] M.C.Bento, O.Bertolami and A.A.Sen, Phys.Rev. **D67** (2003) 063511, hep-ph/0208124.
- [11] D.S.Goldwirth, Phys.Lett. **B243** (1990) 41;  
R.Brandenberger, G.Geshnizjani and S.Watson, Phys.Rev. **D67** (2003) 123510, hep-th/0302222.
- [12] D.S.Salopek and J.R.Bond, Phys.Rev. **D42** (1990) 3936;  
A.G.Muslimov, Class.Quant.Grav. **7** (1990) 231;  
J.E.Lidsey, Phys.Lett. **B273** (1991) 42;  
A.R.Liddle, P.Parsons and J.D.Barrow, Phys.Rev. **D50** (1994) 7222, astro-ph/9408015.
- [13] A.Sen, Mod.Phys.Lett. **A17** (2002) 1797, hep-th/0204143.
- [14] M.Sami, P.Chingangbam and T.Qureshi, Phys.Rev. **D66** (2002) 043530, hep-th/0205179;  
Z.K.Guo, Y.S.Piao, R.G.Cai and Y.Z.Zhang, Phys.Rev. **D68** (2003) 043508, hep-ph/0304236.
- [15] G.N.Felder, A.Frolov, L.Kofman and A.Linde, Phys.Rev. **D66** (2002) 023507.